Progression in Methods of Calculation

A SINGAPORE MATHS APPROACH

St. Cuthbert's RC. Primary School

ST. MARY'S RC. PRIMARY SCHOOL

INTRODUCTION

The National Curriculum emphasises the importance of all pupils' mastery of the content taught each year and discourages the acceleration of pupils into content from subsequent years.

In line with the new curriculum objectives, we will be teaching our children to:

- Become **fluent** in the fundamentals of mathematics, with increasingly complex problems over time so pupils develop conceptual understanding.
- **Reason** mathematically by following a line of enquiry, focusing on explanations and justifications of how they know an answer.
- **Solve problems** by applying mathematics to a variety of problems.

WHAT IS 'SINGAPORE' MATHS?_

Singapore maths is just maths! It is a style of maths teaching that is proven to develop a deeper conceptual understanding of mathematical concepts, giving children opportunities to use and
apply their knowledge. It is child led, with a focus on questioning and discussion where children explore and investigate key mathematical concepts through problem solving.

CONCRETE – PICTORIAL – ABSTRACT (CPA APPROACH)

We will be following a Concrete, Pictorial, Abstract approach throughout the school. Children will begin learning through the use of concrete materials (Base Ten, counters, multilink, tens frames) before progressing onto a pictorial representation of the resources. Children will move onto the abstract approach only when they are secure in their understanding.



TOOLKITS

Children will have the opportunity to access a range of concrete manipulatives in every lesson to help secure their understanding.



Maths boxes in classrooms contain a range of resources such as:

- Number fans
- Numicon
- Place Value cards
- Counters
- Square Tiles
- Digit cards
- Place Value Counters
- A range of dice
- Place Value sliders



- Unifix cubes
- Bead strings
- Base Ten
- Place Value Mats
- Hundred Squares
- Multiplication Grids
- Fraction Walls
- Tens Frames
- Everyday counting tools e.g. dinosaurs, fish etc







MATHEMATICAL LANGUAGE

To ensure consistency across the whole school, it is important we are all using the correct mathematical terminology.

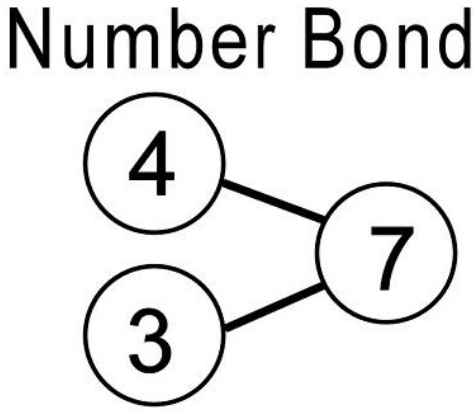
Use of correct mathematical terminology is critical to teaching mathematical vocabulary. Placing math terms on a math word wall and using them daily makes ongoing review easy and fun.

The best way to learn a language - any language- is to be fully immersed in it. And so it should be in the math classroom because math is a language. Every math classroom should be rich with language, and the use of the correct terms must not just be encouraged. It must be expected.

In order for children to learn to speak maths, teachers must model the language regularly and correctly.

NUMBER BONDS

Children must have a secure understanding of number bonds as this will help them when approaching addition and subtraction problems.

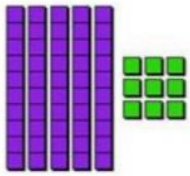


This way children learn the various parts that make the whole.

They recognise that these are interchangeable yet give the same whole.

This then prepares them for the acquisition of addition/subtraction facts which in turn leads to formal algorithm and mental strategies.

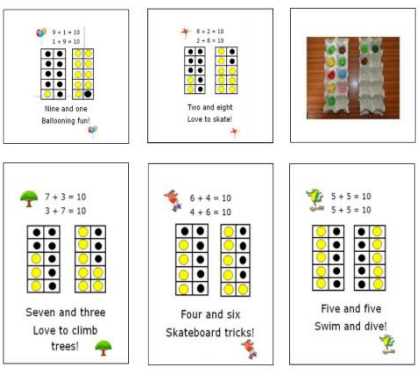
They will need to explore number patterns and relationships using concrete resources such as those contained in the classroom toolkits. Once children have explored making different totals they begin to forge relationships between bonds to 10. They use a number of concrete tools to support this process.




Ten Frames/Egg boxes feature heavily and represent the base 10. Children explore and learn the number of different ways to make 10 using the egg box, base 10 frame.

In EYFS, children learn and explore the different ways of making ten through the use of ten frames and rhymes.



BAR MODEL

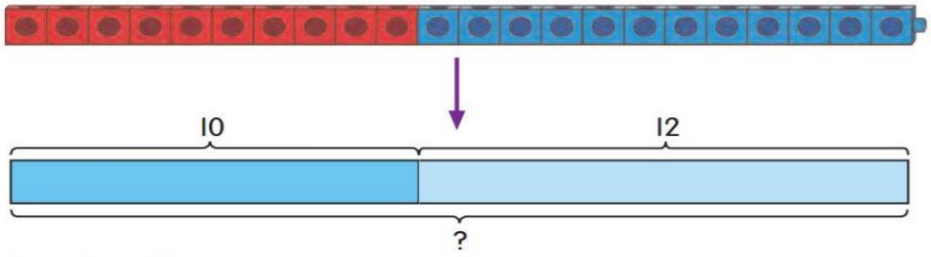
The bar model is a step by step method that helps children to understand and extract the information within a calculation or word problem. By drawing a bar model, children translate a calculation or word problem into a picture. The approach helps children process the information given in the problem, visualise the structure, make connections and solve the problem.

In Year 1 children have been prepared for the introduction of the bar model by using concrete apparatus; for example, using interlocking cubes to compare the number of objects in two groups.

In Year 2 children explore addition and subtraction initially with concrete apparatus before moving on to using a pictorial representation – the bar model.

Adding sets of objects

1 Omar bakes 10 biscuits.
Ruby bakes 12 biscuits.
How many biscuits do they bake altogether?

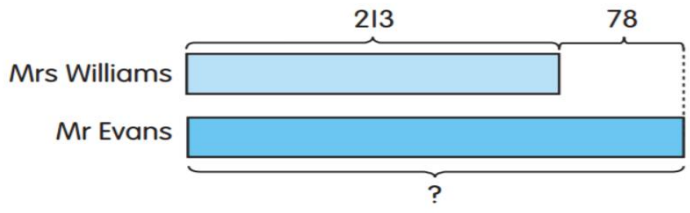


10 + 12 = 22

They bake 22 biscuits altogether.

Comparing two sets

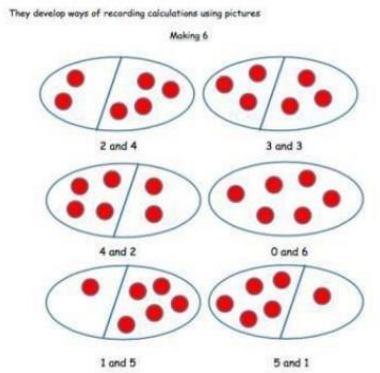
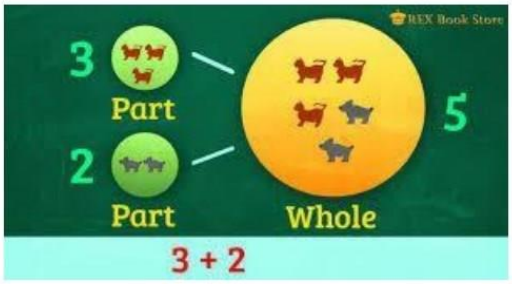
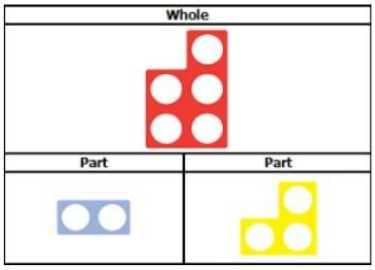
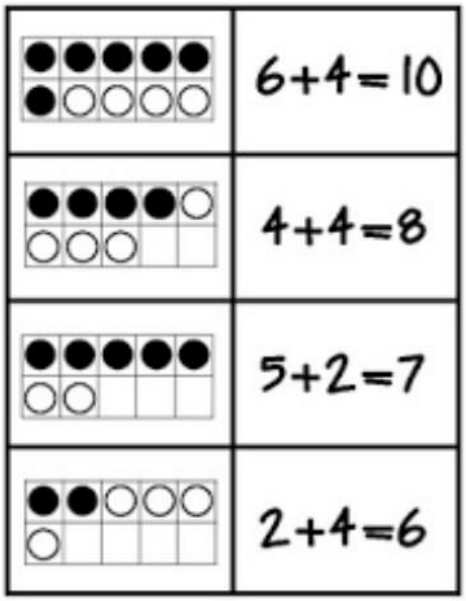
1 Mrs Williams has 213 chickens on her farm.
Mr Evans has 78 more chickens on his farm.
How many chickens does Mr Evans have on his farm?

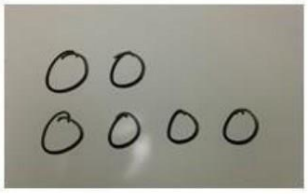


213 + 78 = 291

Mr Evans has 291 chickens on his farm.

Addition (Reception)

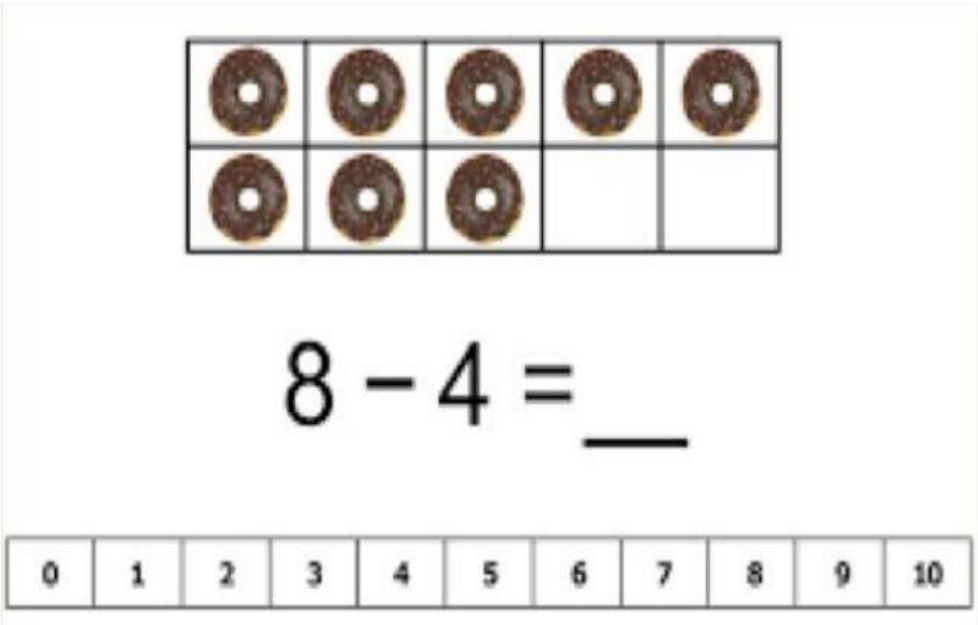
Explore part part whole relationship— combining two parts to make a whole.	  
Using the ten frame/egg boxes to support addition of single digits— counting all/ combining two groups	

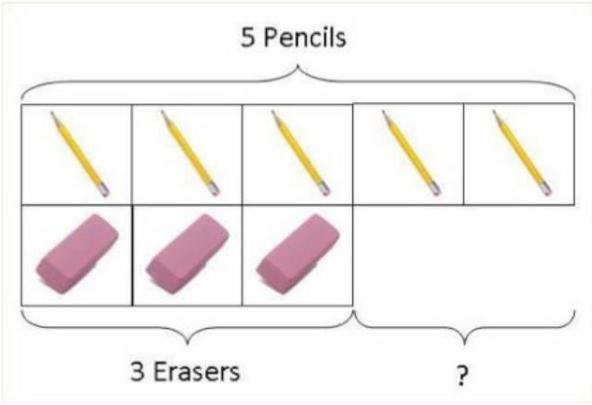
Solving problems using concrete, pictorial images.	Sara has 2 apples. Jon has 5 apples. How many apples do they have altogether? How many more apples does Jon have than Sara?     

Reception

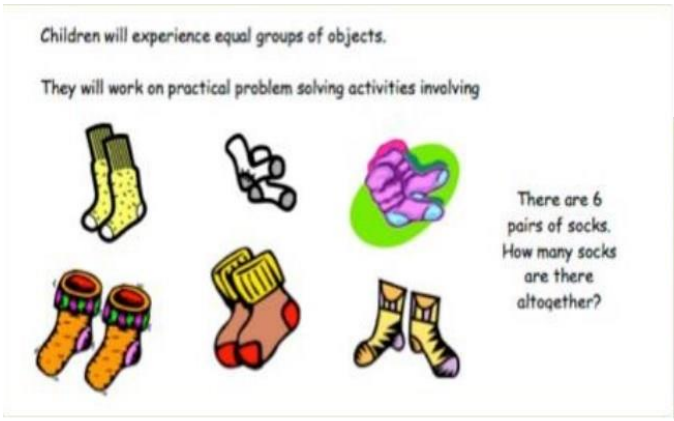
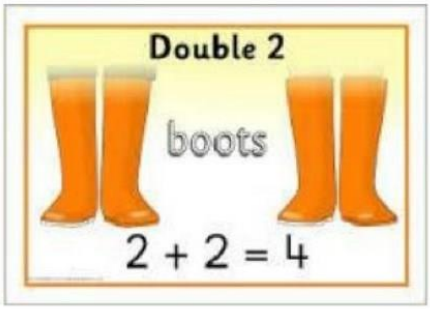
<table>
<tr><th colspan="2">Subtraction (Reception)</th></tr>
<tr><td>Using concrete strategies for counting</td><td>Taking away after counting out practical equipment. . Children would be encouraged to physically remove these using touch counting.


By touch counting and dragging in this way, it allows children to keep track of how many they are removing so they don't have to keep recounting. They will then touch count the amount that are left to find the answer.

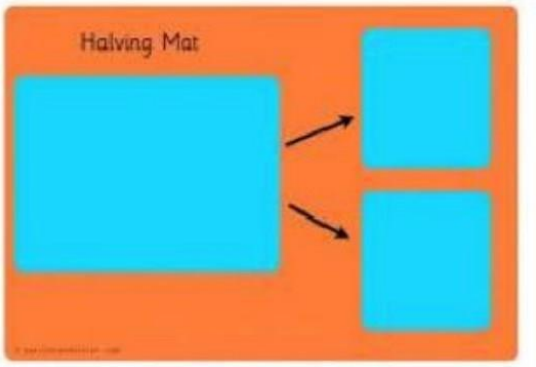
Those who are ready may record their own calculations</td></tr>
<tr><td>Using the ten frames to support subtraction by taking away</td><td>
</td></tr>
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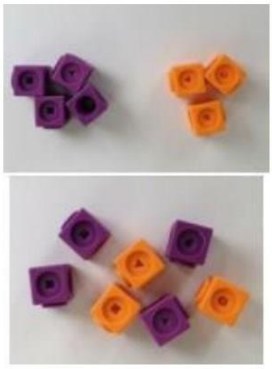
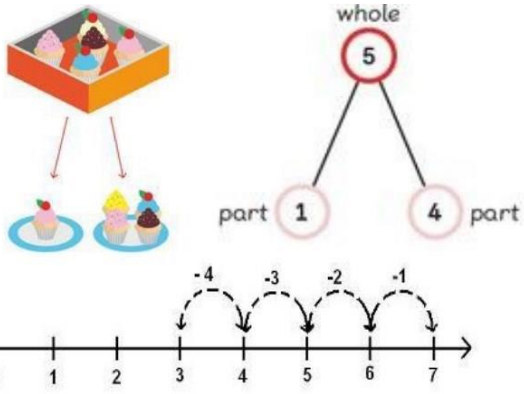
Solving problems using concrete, pictorial images.	Peter has 5 pencils and 3 erasers. How many more pencils than erasers does he have? 

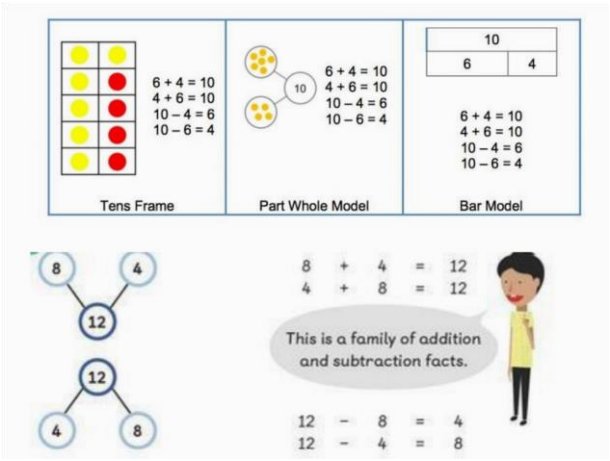
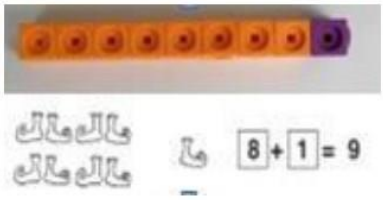
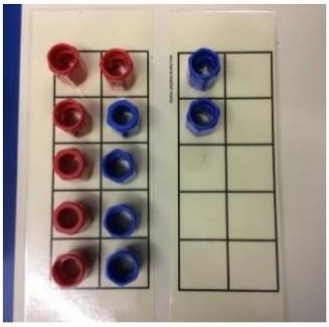
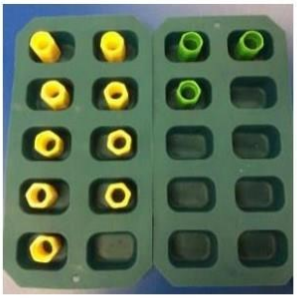
Multiplication (Reception)

Experiencing equal groups of objects They will think about doubling when solving practical problems.	 

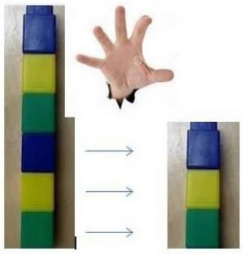
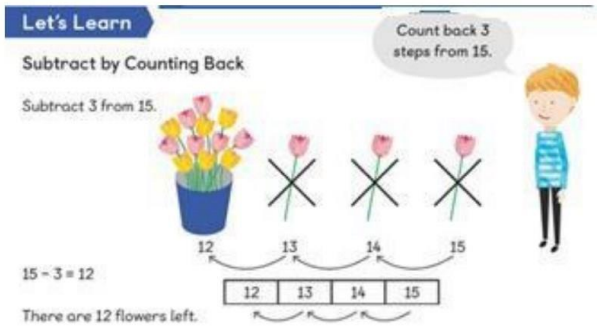
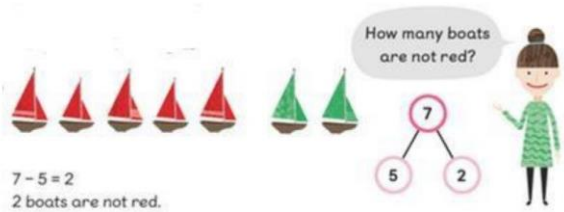
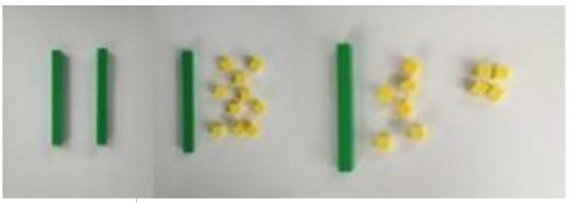
Division (Reception)

Sharing practical objects. Hearing and being exposed to language to describe half and seeing visual representations.	 

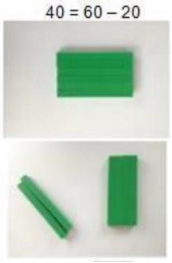
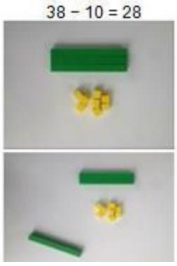
Addition (Year 1)	
Combining two parts to make a whole: part whole model. Joining two groups and then recounting all objects (lots of practice making 10 and numbers to 10 e.g. 6 + 4 = 10 or 3 + 5 = 8)	3 + 4 = 7  

Number Bonds Learn number bonds to 20 and demonstrate related facts. Addition and subtraction taught alongside each other as pupils need to see the relationship between the facts.	
Add and subtract one digit numbers and two digit numbers to 20, including zero	8+1=9 
Bridging 10; 6 + 6 = 12 Make 9 in one and 3 in the other. Take one from the 3 to make the 9 into a ten….10+2 = 12 use ten frames, Singapore bars, egg boxes and number lines to practice. Children should start with the larger number and add the smaller number seeing what	 6 + 6 = 12  Make 9 in one and 3 in the other. Take one from the 3 to make the 9 into a ten….10+2 = 12

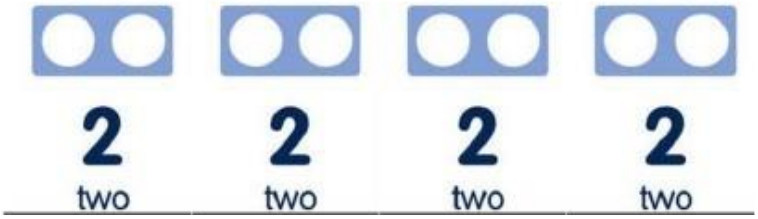
Year 1

Subtraction (Year 1)	
Taking away should begin with physical objects: counters, cubes, Dienes etc	 6 -3=3
Subtraction by counting back	
Subtracting a single digit number from a single digit number and a single digit from a two digit by crossing out pictures	
Subtracting using the part part whole model (include problem solving with missing digits). ? - 5 = 2	
When subtracting using Dienes children should be taught to regroup (rename) a ten rod for 10 ones and then subtract from those	 20 – 4 = 16

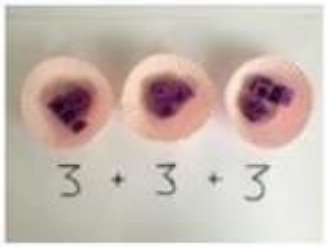
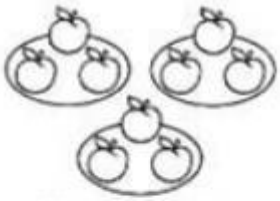
Year 1

Subtracting Multiples of 10. Using the vocabulary of 1 ten, two tens, etc, alongside 10, 20, 30 is important	 

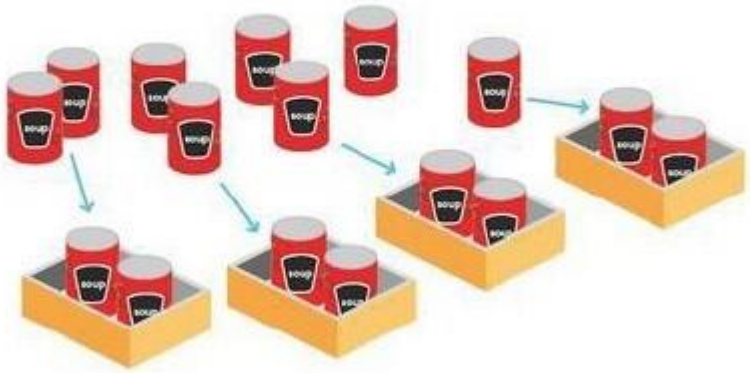
Multiplication (Year 1)

Counting in Multiples of 2, 5 and 10 from zero. Children should count the number of groups on their fingers as they are skip counting.	2 4 6 8 4 groups of 2 = 8 4 x 2 = 8  2 × 4 = 8 
When moving to pictorial/written calculations the language is important	 This image represents two groups of 4 or 4 twice

Year 1

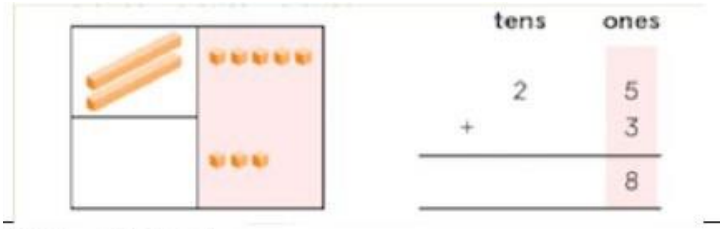
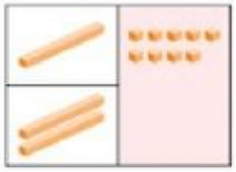
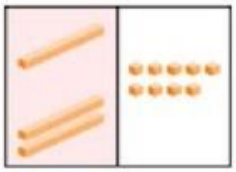
Solving Multiplication Problems using repeated addition	  How many apples are there altogether? 3 + 3 + 3 = 9

Division (Year 1)

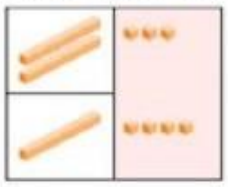
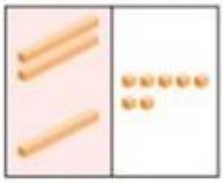
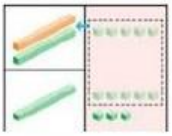
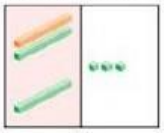
Pupils should be taught to divide by working practically and the sharing should be shown below the whole to familiarise children with the concept of the whole.	10 ÷ 2 = 5 1 There are 8 cans.  There are 4 boxes of 2 cans.

Year 1

Year 2

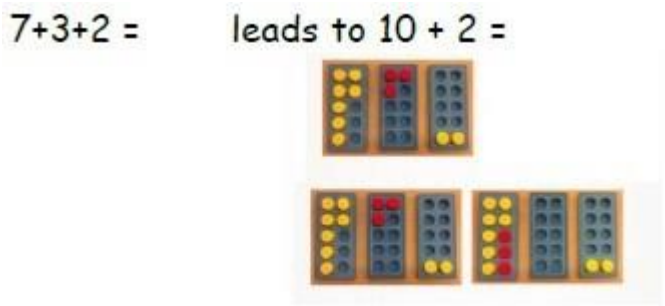
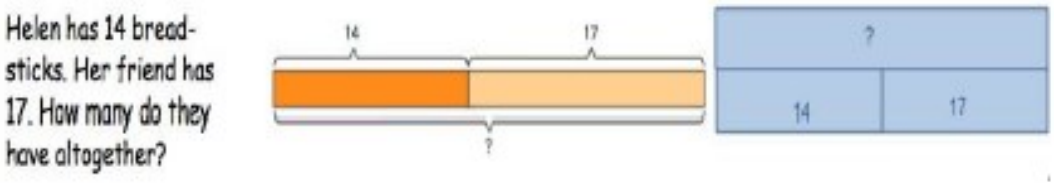
Addition (Year 2)	
Using concrete and pictorial representations to add a 2 digit number to a 1 digit number and a 2 digit number to a tens number.	 Step 1 Add the ones.  Step 2 Add the tens. 1 ten + 2 tens = 3 tens  19 + 20 = 39

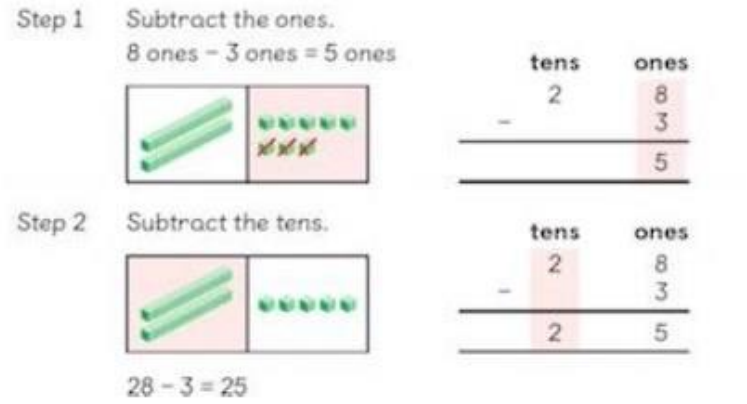
Year 2

Using concrete and pictorial representations to add two 2 digit numbers.	Step 1 Add the ones. 3 ones + 4 ones = 7 ones  tens ones 2 3 + 1 4 7 Step 2 Add the tens. 2 tens + 1 ten = 3 tens  tens ones 2 3 + 1 4 3 7 23 + 14 = 37 Adding with renaming Add 15 and 18. Use  to help you add.  Step 1 Add the ones. 5 ones + 8 ones = 13 ones Regroup the ones. 13 ones = 1 ten and 3 ones  tens ones 1 5 + 1 8 1 3 Step 2 Add the tens. 1 ten + 1 ten + 1 ten = 3 tens  tens ones 1 5 + 1 8 1 3 + 2 0 3 3 15 + 18 = 33

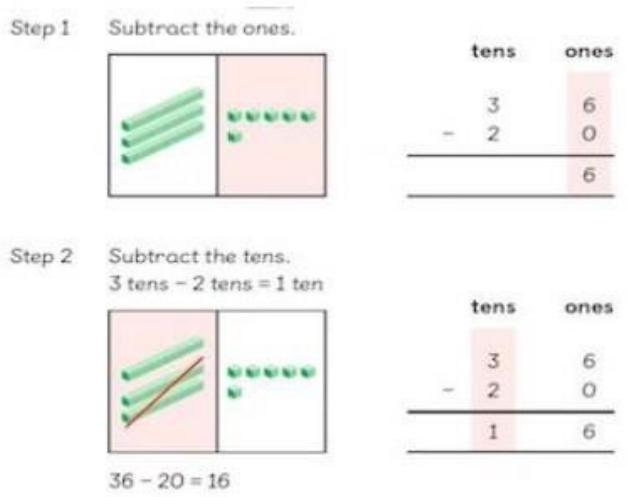
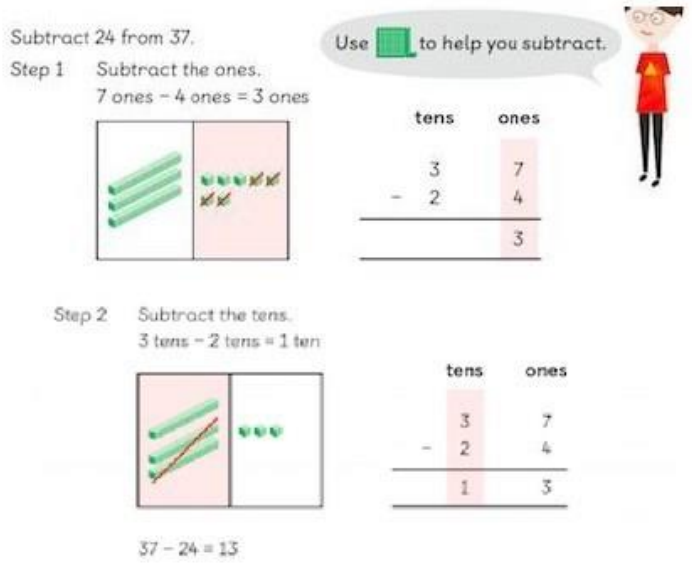
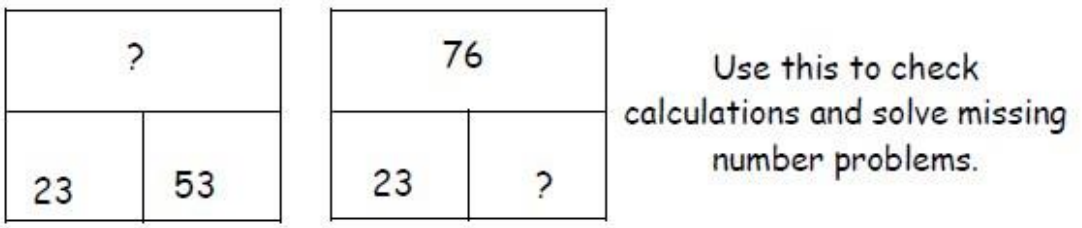
Addition (Year 2)

Year 2

Using concrete and pictorial representations to add 3 single digit numbers.	
Using the bar model to find missing digits: It is important for the children to use the bar model in this way to encourage the use of it to aid problem solving.	

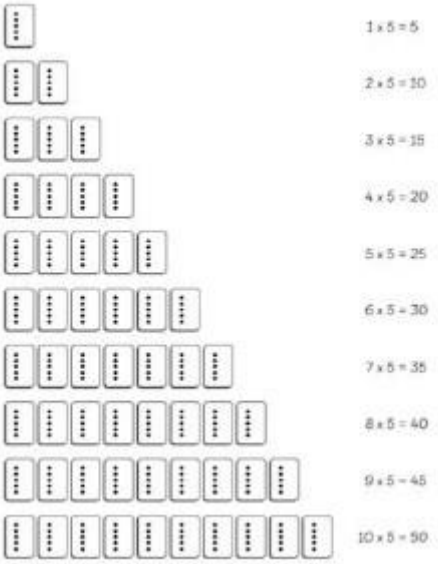
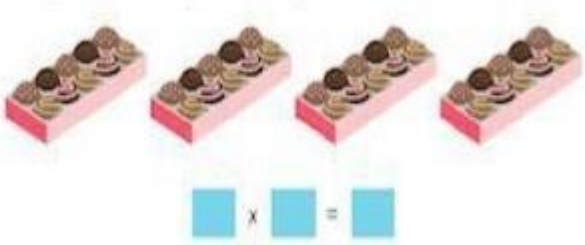
Subtraction (Year 2)	
Using concrete and pictorial representations to subtract a 1 digit number from a 2 digit number	

Year 2

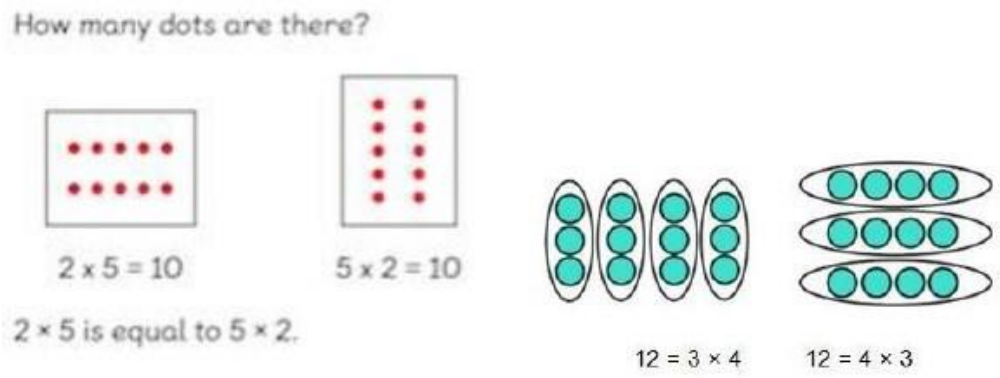
Using concrete and pictorial representations to subtract a 2 digit number from a tens number	
Using concrete and pictorial representations to subtract a 2 digit number from a 2 digit number	
Recognise and use the inverse relationship between addition and subtraction.	

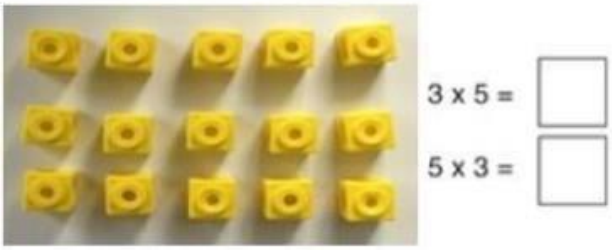
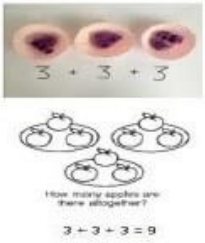
Multiplication (Year 2)

Year 2

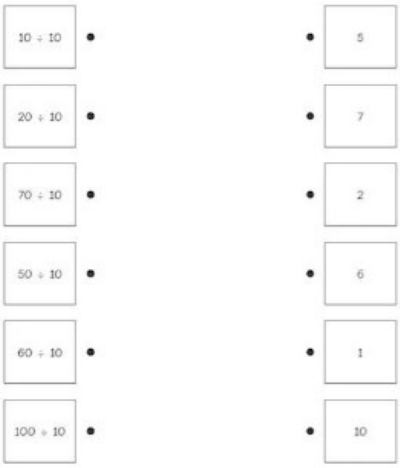
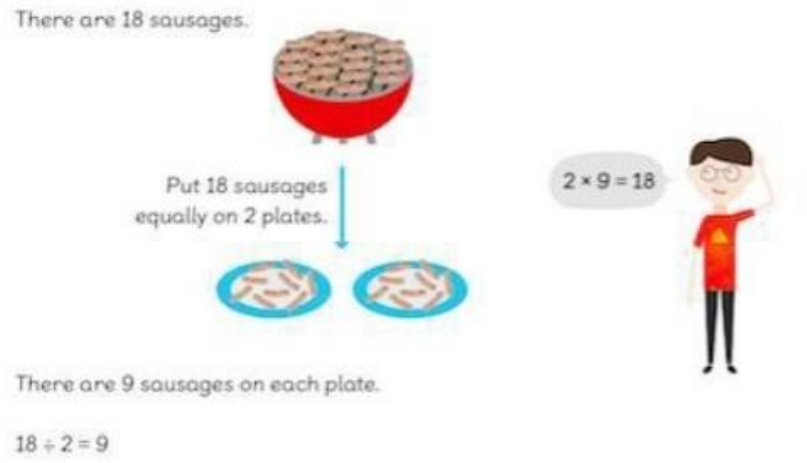
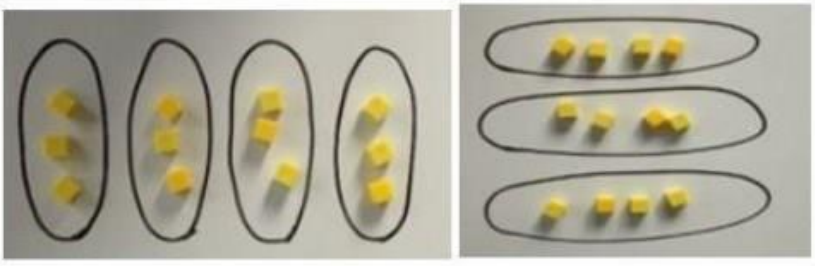
Skip counting in multiples of 2, 3, 5 and 10 from zero.	
Recall and reuse multiplication facts for the 2, 5 and 10 times tables.	
Use multiplication sign (X) and equals sign (=) when writing out multiplication tables.	

Year 2

Understand that multiplication is commutative Pupils should understand that an array can represent different equations and that as multiplication is commutative the order doesn't affect the answer.	

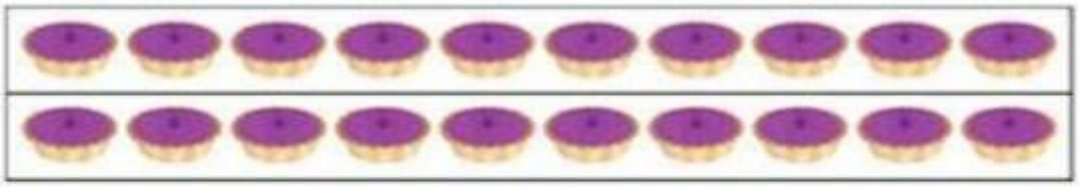
Multiplication (Year 2)	
Solve multiplication problems using arrays and repeated addition.	 

Year 2

Division (Year 2)	
Recall and use the division facts for 2, 5 and 10 multiplication tables.	
Solve division problems in context by using concrete objects by sharing.	
Solve division problems in context using arrays.	

Year 2

Solve division using grouping.	Put 10 buns in groups of 2. How many plates are there?   Put into groups of 5. There are ☐ groups.

Division (Year 2)	
Use the inverse This should be taught alongside both multiplication and division.	Make a family of multiplication and division facts.  2 x 10 = 20 ——— 20 ÷ 10 = ☐ 10 x 2 = 20 ——— 20 ÷ 2 = ☐

Year 3

Addition (Year 3)

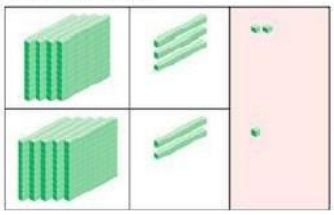
Add two three digit numbers.

Children need to first use equipment to support understanding of place value.

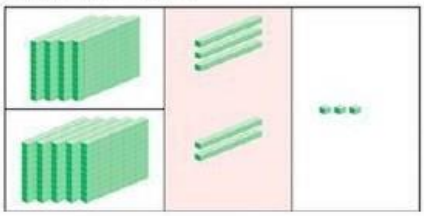
Start without renaming then gradually move onto renaming.

432 + 521 =

Step 1 Add the ones.
2 ones + 1 one = 3 ones

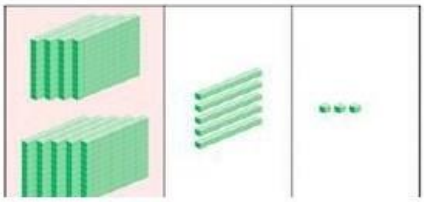


Step 2 Add the tens.
3 tens + 2 tens = 5 tens



	h	t	o
	4	3	2
+	5	2	1
		5	3

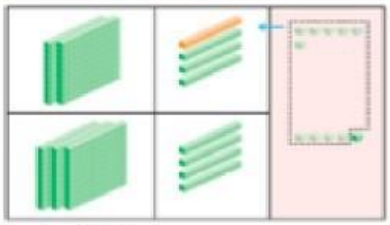
Step 3 Add the hundreds.
4 hundreds + 5 hundreds = 9 hundreds



	h	t	o
	4	3	2
+	5	2	1
	9	5	3

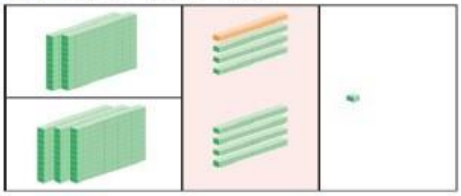
	h	t	o
	4	3	2
+	5	2	1
			3

236 + 345 =



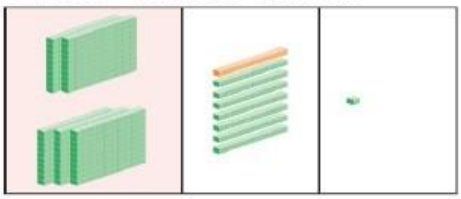
	h	t	o
	2	3	6
+	3	4	5
			1

Step 2 Add the tens.
1 ten + 3 tens + 4 tens = 8 tens



	h	t	o
	2	3	6
+	3	4	5
		8	1

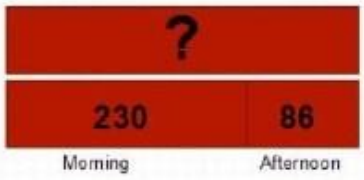
Step 3 Add the hundreds.
2 hundreds + 3 hundreds = 5 hundreds



	h	t	o
	2	3	6
+	3	4	5
	5	8	1

236 + 345 = 581

Year 3

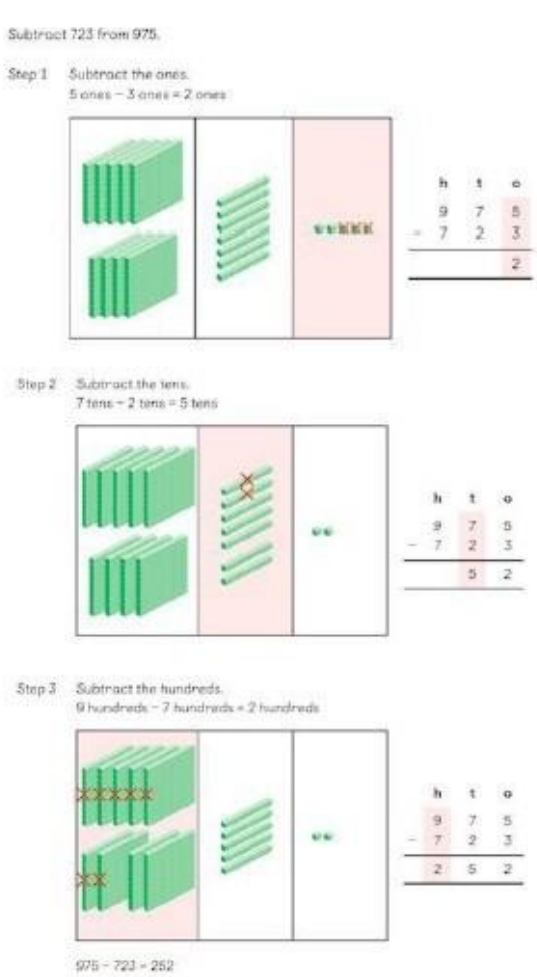
Bar Modelling It is important for the children to use the bar model in this way to encourage the use of it to aid problem solving.	**Bar Model to support understanding of problem solving:**  A man sold 230 balloons at a carnival in the morning. He sold another 86 balloons in the evening . How many balloons did he sell in all? 

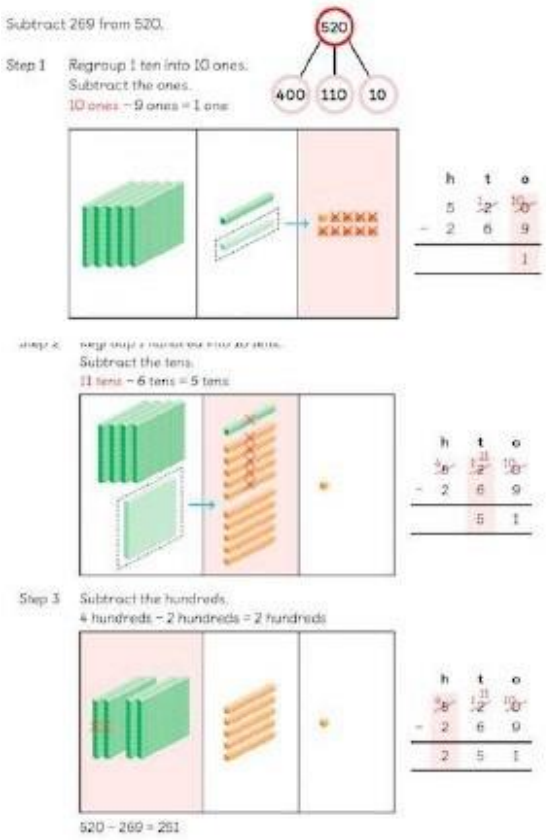
Subtraction (Year 3)

Subtract up to 3 digits from 3 digits.

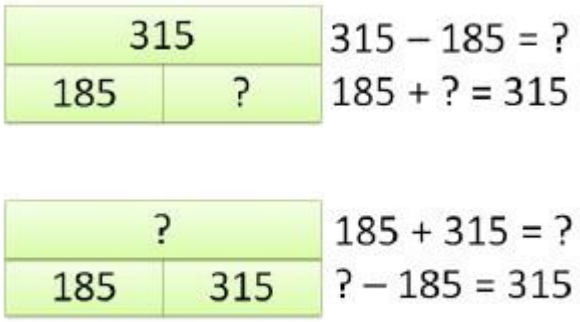
Children need to first use equipment to support understanding of place value.

Only when children are secure with method should exchanging be introduced.





Year 3

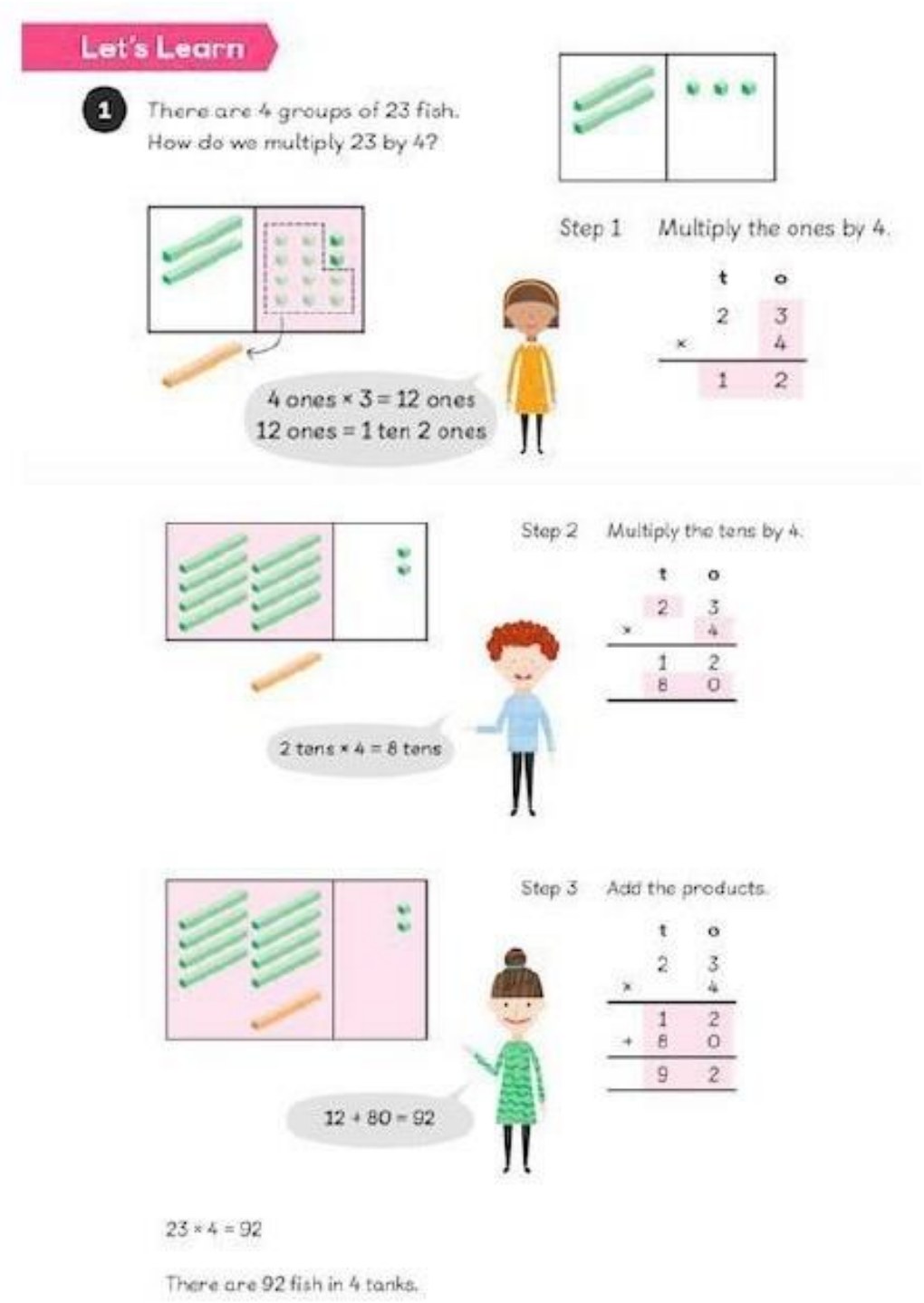
Bar Modelling It is important for the children to use the bar model in this way to encourage the use of it to aid problem solving.	

Multiplication (Year 3)

Year 3

Children should be able to recall the 2, 5, 10, 3, 4 and 8 multiplication tables.

Multiply a 2 digit number by a 1 digit number.



Bar Modelling

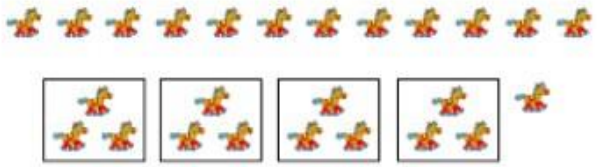
It is important for the children to use the bar model in this way to encourage the use of it to aid problem solving.

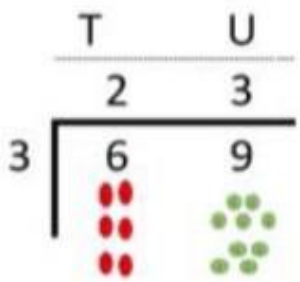
4 children go to the cinema. They each pay £15. How much do they spend altogether?

Whole unknown

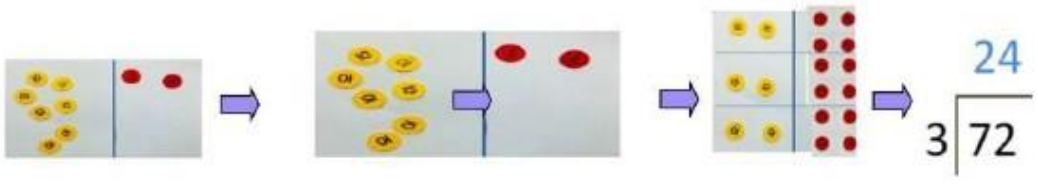


Year 3

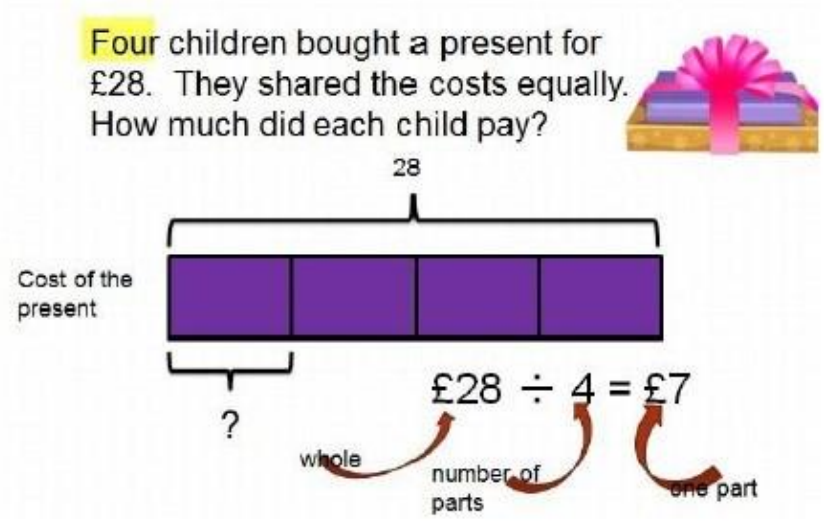
<table>
<tr><th colspan="2">Division (Year 3)</th></tr>
<tr><td>Dividing and grouping understanding the concept of remainders.</td><td>Start with using the real objects-or objects that represent the calculation.

13 ÷ 4 = 3 Remainder 1</td></tr>
<tr><td>Dividing using short division

Once the children are secure with division as grouping and can demonstrate this on number lines, arrays etc. short division should be introduced for dividing larger 2 digit numbers. Initially with carefully chosen calculations requiring no remainders. Compare the lay out of short division o that of an array.</td><td>

Remind children of correct place value, that 69 is equal to 60 and 9, but in short division, pose:
· How many 3's in 6? = 2, and record it above the 6 tens.
· How many 3's in 9? = 3, and record it above the 9 ones.

Once children demonstrate a full understanding of remainders, and also the short division method taught, they can be taught how to use the method when remainders occur within the calculation (e.g. 72÷3), and be taught to 'carry' the remainder onto the next digit.

</td></tr>
</table>

Year 3

Bar Modelling It is important for the children to use the bar model in this way to encourage the use of it to aid problem solving.	

Year 4

Addition (Year 4)

Adding numbers with up to 4 digits.

Again this should start with the children using equipment to support and lots of discussion about the values of digits.

$$\begin{array}{r} 2\ 3\ 1\ 4 \\ +\ 4\ 2\ 4\ 0 \\ \hline 6\ 5\ 5\ 4 \end{array}$$

Step 1 Add the ones.
4 ones + 0 ones = 4 ones

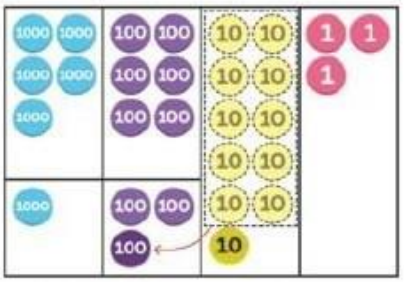
Step 2 Add the tens.
1 tens + 4 tens = 5 tens

Step 3 Add the hundreds.
3 hundreds + 2 hundreds = 5 hundreds

Step 4 Add the thousands.
2 thousands + 4 thousands = 6 thousands

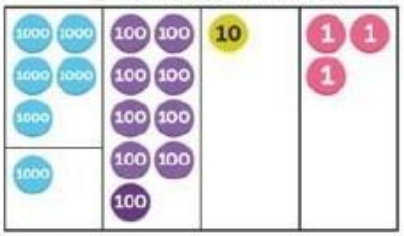
2314 + 4240 = 6554

Step 2 Add the tens. 7 tens + 3 tens + 1 ten = 11 tens
Rename the tens. 11 tens = 1 hundred and 1 ten



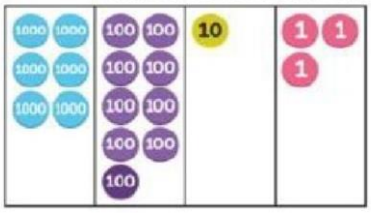
$$\begin{array}{r} 5\ \overset{1}{6}\ \overset{1}{7}\ 8 \\ +\ 1\ 2\ 3\ 5 \\ \hline \quad\quad 1\ 3 \end{array}$$

Step 3 Add the hundreds.
6 hundreds + 2 hundreds + 1 hundred = 9 hundreds



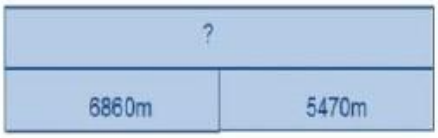
$$\begin{array}{r} 5\ \overset{1}{6}\ \overset{1}{7}\ 8 \\ +\ 1\ 2\ 3\ 5 \\ \hline \quad\quad 9\ 1\ 3 \end{array}$$

Step 4 Add the thousands.
5 thousands + 1 thousand = 6 thousands



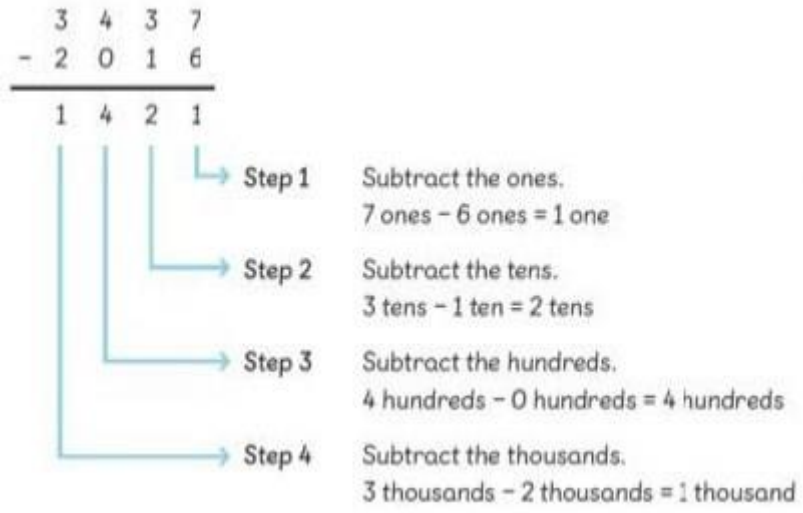
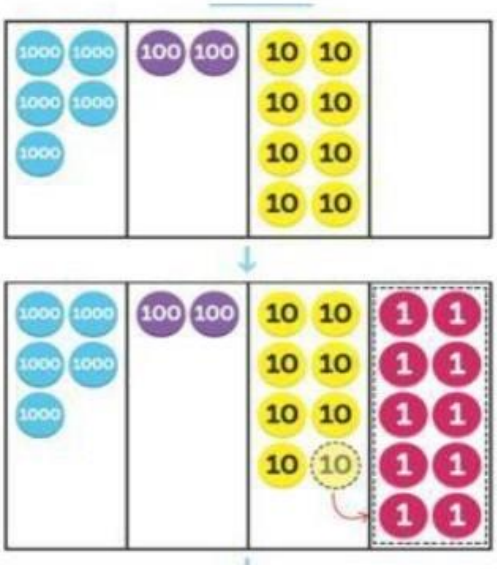
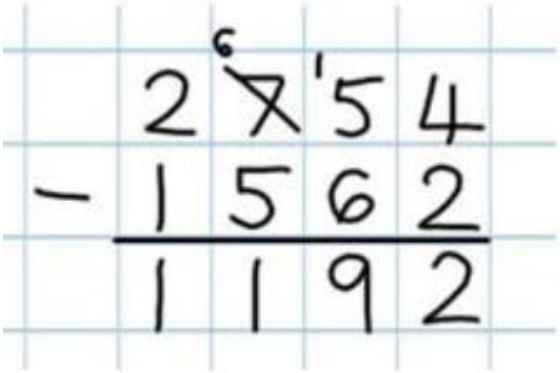
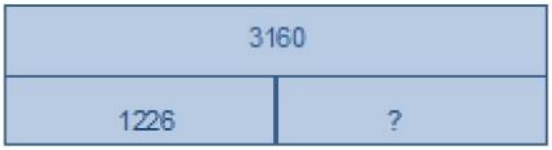
$$\begin{array}{r} 5\ \overset{1}{6}\ \overset{1}{7}\ 8 \\ +\ 1\ 2\ 3\ 5 \\ \hline 6\ 9\ 1\ 3 \end{array}$$

Year 4

Using the bar model to find missing digits.	This is not a form of getting the correct answer but helping to guide children to the correct operation. Alison jogs 6,860 metres and Calvin jogs 5,470 metres. How far do they jog altogether? 

Subtraction (Year 4)

Year 4

Subtract with numbers up to four digits, including exchanging.	   5 2 7 10 - 3 1 6 9 
Using the bar model to find missing digits.	There are 3,160 books in a shop. 1,226 are in English and the rest are in French. How many French books are there? 

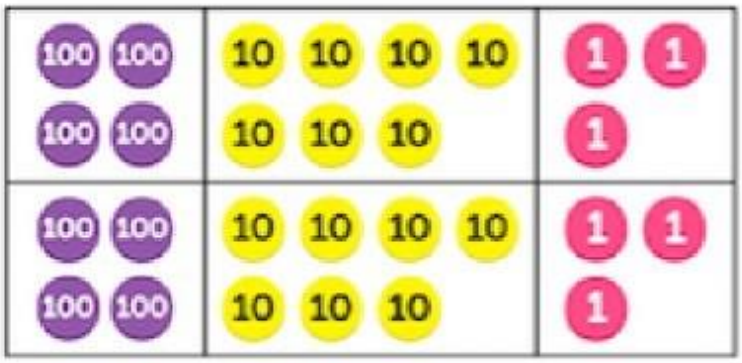
Year 4

Multiplication (Year 4)

Children know all times tables up to 12 x 12.

Children use expanded column multiplication

$$
\begin{array}{rl}
 & 314 \\
 & \times\ 3 \\
\hline
 & 12 \quad (3 \times 4) \\
 & 30 \quad (3 \times 10) \\
+ & 900 \quad (3 \times 300) \\
\hline
 & 942
\end{array}
$$



$$
\begin{array}{rrrr}
 & 4 & 7 & 3 \\
\times & & & 2 \\
\hline
 & & & \\
\hline
\end{array}
$$

Multiply using the bar model

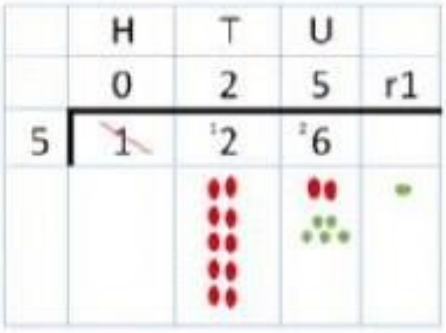
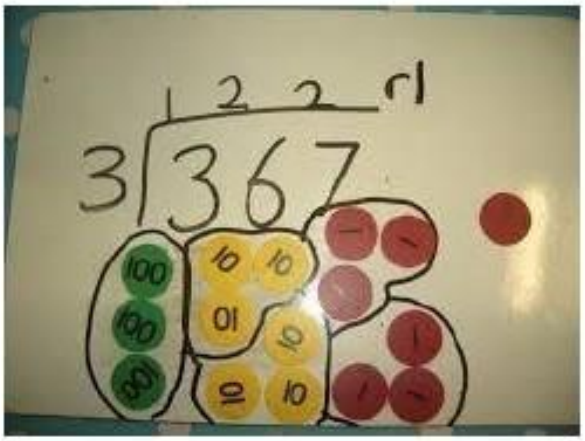
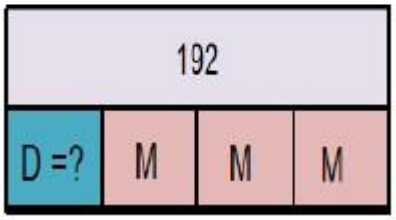
A computer costs 5 times as much as a television. The television costs £429.

How much does the computer cost?



Division (Year 4)

Year 4

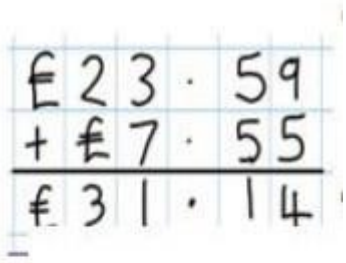
Dividing 3 digit numbers by a 1 digit number using short division.	 
Divide using the bar model	Desmond and Melissa collect cards. They have 192 cards in all. Melissa has three times as many cards as Desmond. How many cards does Desmond have? 

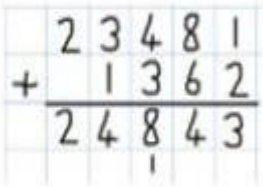
Year 5

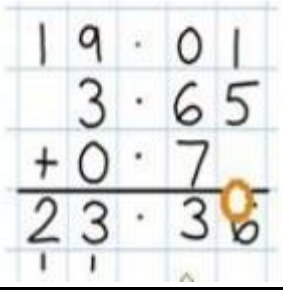
Addition (Year 5)

Adding numbers with more than 4 digits including decimals.

Using place value charts and place value counters is key when understanding adding decimals.



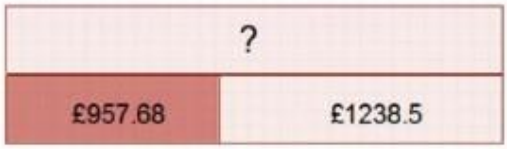




Using the bar model to find missing digits.

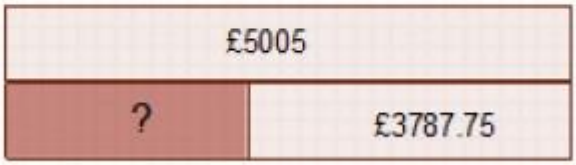
This is not a form of getting the correct answer but helping to guide children to the correct operation.

MacDonalds sold £9957.68 worth of hamburgers and £1238.5 worth of chicken nuggets. How much money did they take altogether?



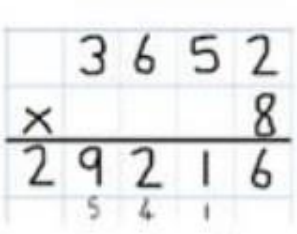
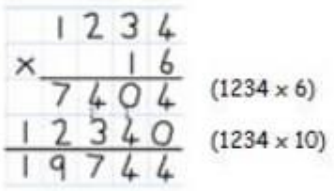
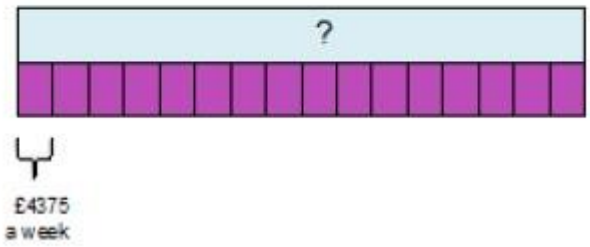
Subtraction (Year 5)

Year 5

Subtract with at least 4 digit numbers including two decimal places. Include money, measures and decimals.	Subtract with decimal values, including mixtures of integers and decimals, aligning the decimal point. $\begin{array}{r} 2\not{3}\,{}^{1}\not{0}\,{}^{1}0\,{}^{1}\not{5}\,{}^{1}6 \\ -\quad 2\,1\,2\,8 \\ \hline 2\,8\,9\,2\,8 \end{array}$ $\begin{array}{r} {}^{6}\not{7}\,{}^{1}\not{0}\,{}^{8}\not{6}\,{}^{1}\not{9}\cdot{}^{1}0 \\ -\quad 3\,7\,2\cdot 5 \\ \hline 6\,7\,9\,6\cdot 5 \end{array}$
Using the bar model to find missing digits.	A whole to Lapland costs £5005 for a family of four, the Smith's have only saved £3787.75, how much money do they still need to find? 

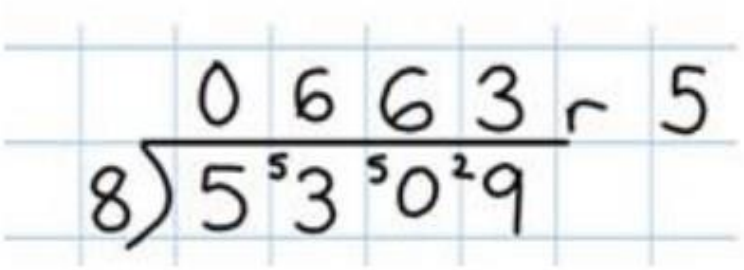
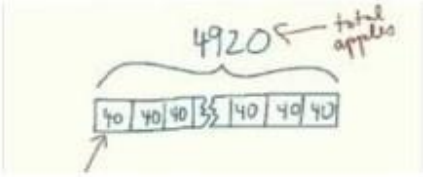
Multiplication (Year 5)

Year 5

Multiply up to 4 digit numbers by 2 digit numbers using long division. Children need to be taught to approximate first to check the reasonableness of their answers. So 56 x 27 could be 60 x 30 = 1800	56 X 27 392 (56x7) 1120 (56x20) 1512 Explain that first we are multiplying the top number by 7 starting with the units. (any carrying needs to be done underneath the numbers). · Now explain that we need to put a 0 underneath—explain that this is because we are multiplying the number by 20.. (2 tens) which is the same as multiplying 10 and 2. · Now add the 2 numbers together to give you the answer. · This will need lots of modeling to show the children.  
Using the bar model to support multiplication	The cost to run a sports centre is £4375 a week, how much would it cost to run for 16 weeks? 

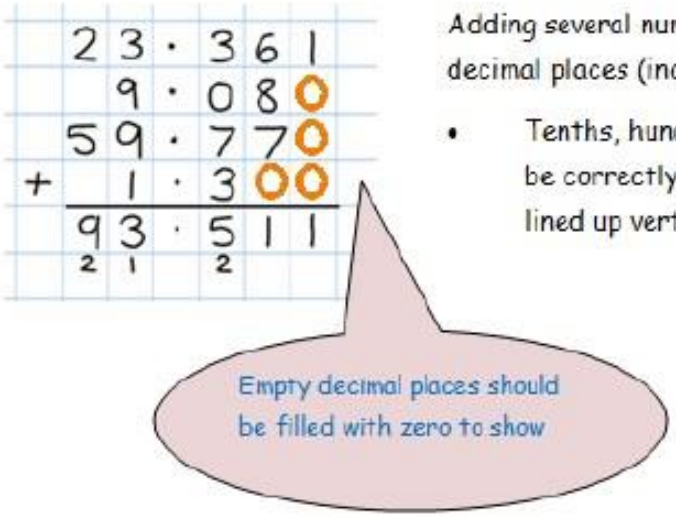
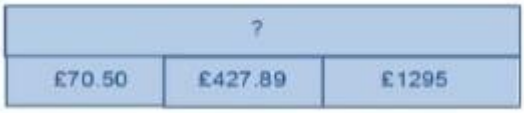
Division (Year 5)

Year 5

Divide up to 4 digit numbers by 1 digit numbers using short division. Also numbers that have remainders.	 **Short division with remainders:** Now that pupils are introduced to examples that give rise to remainder answers, division needs to have a real life problem solving context, where **pupils consider the meaning of the remainder and <u>how</u> to express it**, ie. as a fraction, a decimal, or as a rounded number or value , depending upon the context of the problem.
Using the bar model to support division	**<u>Bar Model to support understanding of problem solving:</u>** Frank has 4920 apples. He needs to put them into baskets of 40. How many baskets does he need? 

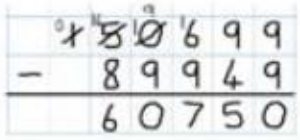
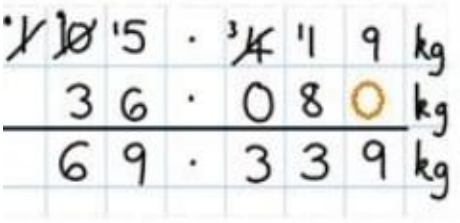
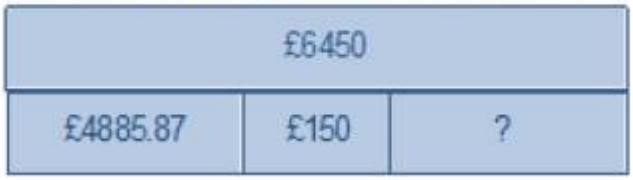
Year 6

Addition (Year 6)

Adding several numbers with up to 3 decimal places.	 Adding several numbers with different numbers of decimal places (including money and measures): • Tenths, hundredths and thousandths should be correctly aligned, with the decimal point lined up vertically including in the answer row.
Using the bar model to support addition	Jack went on holiday. His flight cost £70.50, the hotel £1295 and spending money £427.89. How much did Jack spend on his holiday? 

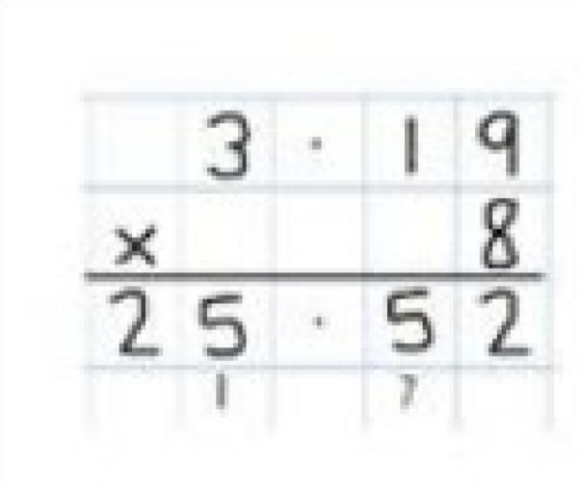
Subtraction (Year 6)

Year 6

Subtracting with increasingly more complex numbers including decimals.	Very important to use in a range of contexts- measures and money.  
Using the bar model to support subtraction	Chloe wants to buy a new car for £6450. She has £4885.87 in her savings account. Her Dad gives her £150 for her birthday. How much more money does she need to save? 

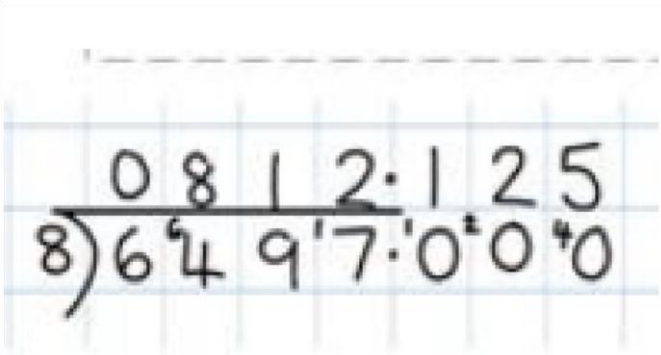
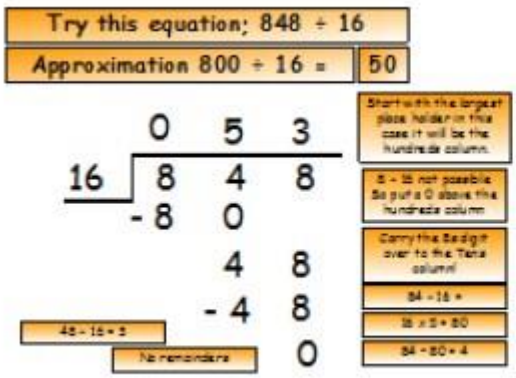
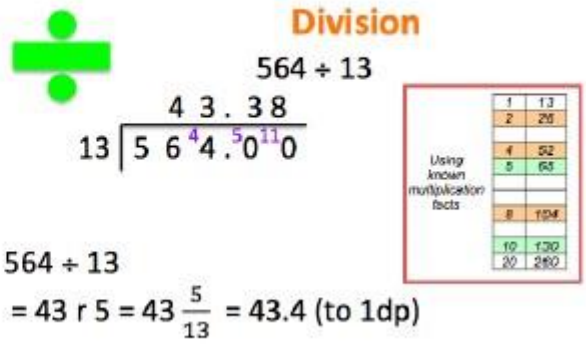
Multiplication (Year 6)

Year 6

Short and long multiplication with up to 2 decimal places.	
Using the bar model to support multiplication	If 5 friends went on holiday and each paid £579.75 what was the total cost of the holiday? 

Division (Year 6)

Year 6

Short division to divide by a 1 digit number.	 **Short division with remainders**: Pupils should continue to use this method, but with numbers to at least 4 digits, and understand how to express remainders as fractions, decimals, whole number remainders, or rounded numbers. Real life problem solving contexts need to be the starting point, where pupils have to consider the most appropriate way to express the remainder.
Long division to divide by 2 digit number.	  564 ÷ 13 = 43 r 5 = $43\frac{5}{13}$ = 43.4 (to 1dp)
Using the bar model to support division	